PREPARATION FOR MATH TESTING

at

CityLab Academy

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Refresh your math skills with a

MATH REVIEW

and find out if you are ready for the math entrance test by taking a

PRE-TEST

(see back page)

Calculators are not allowed during the test, so make sure to practice without a calculator.

To sign up for the math and English tests, call 617-638-5664 or email medcad@bu.edu

Table of Contents

PLEASE NOTE: This math review booklet covers most of the topics and provides examples of the types of problems that are on the math entrance test. Two topics that are not covered are basic algebraic equations and square roots.

Reviews of these topics can be found online at the following websites:

Basic Algebraic Equations
http://library.thinkquest.org/C0110248/algebra/alexpressions.htm
http://www.sosmath.com/algebra/solve/solve0/solve0.html

Square Roots
http://mathforum.org/~sarah/hamilton/ham.squareroots.html
http://regentsprep.org/Regents/math/radicals/Laddsubt.htm
http://www.purplemath.com/modules/radicals.htm

ADDING AND SUBTRACTING DECIMALS

When adding or subtracting numbers with decimal points:

Step 1 **Write the numbers in a column. Line up the decimal points.**

EXAMPLE 1

$3.42 + 5.97 \Rightarrow$

$$\begin{array}{r} 3.42 \\ +\ 5.97 \end{array}$$

Step 2 **For whole numbers, place a decimal point after the number. Then add one or more zeros to the right of the decimal point.**

EXAMPLE 2

$4 + 4.623 \Rightarrow$

$$\begin{array}{r} 4.000 \\ +\ 4.623 \end{array}$$

For this problem, re-write 4 as 4.000

↓

Step 3 **Add (or subtract) the numbers the same way as for numbers without decimal points.**

$$\begin{array}{r} 4.000 \\ +\ 4.623 \\ \hline 8.623 \end{array}$$

Step 4 **Bring the decimal point straight down into the answer.**

PRACTICE

1) $10.5 - 1.29 =$

2) $354.078 + 6.12 + 0.163 =$

3) $3.44 - 0.79 =$

4) $2{,}005.1 + 7.9 + 80.06 =$

5) $13.88 - 0.56 =$

6) $56 + 3.25 + 2.98 =$

ANSWERS

1) 9.21 2) 360.361 3) 2.65 4) 2,093.06 5) 13.32 6) 62.23

MULTIPLYING DECIMALS

When multiplying numbers with decimal points:

		EXAMPLE 1	
Step 1	**Count the number of decimal places (numbers to the right of the decimal point) in all the numbers being multiplied. Add to find the total number of decimal places.**	3.1 x 2.3 ?	1 decimal place + 1 decimal place 2 decimal places
Step 2	**Multiply the numbers as if they were whole numbers without decimal points and record the answer.**	31 x 23 713	
Step 3	**To determine where to put the decimal point in the answer, use the total number of decimal places. Start at the right of the answer and count to the left. Place the decimal point in front of the last number counted.**	3.1 x 2.3 7.13	1 decimal place + 1 decimal place 2 decimal places
		EXAMPLE 2	
Step 4	**If more decimal places are needed, add zeroes in front of the number.**	0.005 x 0.07 0.00035	3 decimal places + 2 decimal places 5 decimal places

To count 5 decimal places, add zeros in front of the 3.

PRACTICE

1) 453 x 0.03

2) 5,751 x 0.006

3) 0.47 x 0.2

4) 0.025 x 22

5) 23.54 x 0.05

6) 45.67 x 0.23

7) 555 x 1.76

8) 950 x 0.087

ANSWERS

1) 13.59 2) 34.506 3) 0.094 4) 0.55 5) 1.177 6) 10.5041 7) 976.8 8) 82.65

DIVIDING DECIMALS

TIP!
Know the parts of the division operation.

Quotient
Divisor ⟌ Dividend

Dividend ÷ divisor = quotient

I. When dividing decimals by <u>whole numbers</u>:

Step 1 **Divide as you would normally (ignoring the decimal point).**

Step 2 **Bring the decimal point straight up into the answer. (In other words, put the decimal point in the quotient directly above the decimal point in the dividend.)**

$$3\overline{)3.9} \quad 1\,3 \longrightarrow 3\overline{)3.9} \quad 1.3$$

II. When dividing decimals by decimals:

Step 1 **First move the decimal point in the divisor to the right to make it a whole number. Then move the decimal point in the dividend the same number of places to the right.**

$$0.16\overline{)0.384} \longrightarrow 0.16\overline{)0.38.4} \longrightarrow 16\overline{)38.4}$$

Move decimal point two places in both divisor and dividend.

Step 2 **Perform the division.**

Step 3 **Bring the decimal point straight up into the answer.**

$$16\overline{)38.4} \quad 2.4$$

PRACTICE

1) $0.5\overline{)300}$ 2) $0.025\overline{)7}$ 3) $5.6\overline{)280}$

4) $0.08\overline{)64.8}$ 5) $1.6\overline{)0.72}$ 6) $63.9 \div 0.009 =$

ANSWERS

1) 600 2) 280 3) 50 4) 810 5) 0.45 6) 7100

EQUIVALENT FRACTIONS

Equivalent fractions are two or more fractions that have the same value.

To determine whether two fractions are equivalent:

EXAMPLE 1

$\frac{12}{24} = \frac{2}{4}$

Step 1 **Look at the fraction with the larger numbers. Determine what number the larger numerator needs to be divided by to give the numerator in the other fraction.**

$\frac{\mathbf{12}}{24} \mathbf{\div 6 =} \frac{\mathbf{2}}{4}$

Step 2 **Divide the denominator of the fraction with the larger numbers by the same number.**

$\frac{12}{\mathbf{24 \div 6 =}} \frac{2}{\mathbf{4}}$

Step 3 **Check to see that the fractions are equivalent: If the numerator and denominator of one fraction can be divided by the same number to give the numerator and denominator of the second fraction, then the fractions are equivalent.**

$\frac{12 \mathbf{\div 6}}{24 \mathbf{\div 6}} = \frac{2}{4}$

EXAMPLE 2:

$\frac{15}{21} = \frac{5}{7}$ but $\frac{8}{22} \neq \frac{4}{7}$ (≠ is the symbol for "not equal to")

because $\frac{15 \div 3}{21 \div 3} = \frac{5}{7}$ but $\frac{8 \div 2}{22 \div 2} \neq \frac{4}{7}$

PRACTICE

Determine whether the following fractions are equivalent.

1) $\frac{15}{25} = \frac{3}{5}$ 2) $\frac{3}{81} = \frac{1}{9}$ 3) $\frac{7}{8} = \frac{49}{56}$ 4) $\frac{2}{13} = \frac{6}{39}$

ANSWERS

1) yes 2) no 3) yes 4) yes

REDUCING FRACTIONS

When the only number that can evenly divide the numerator and denominator of a fraction is 1, the fraction is said to be "reduced to lowest terms" or "simplified".

To reduce a fraction:

EXAMPLE

$\frac{14}{63} \Rightarrow \frac{14 \div 7}{63 \div 7} \Rightarrow \frac{2}{9}$

Step 1 **Divide the numerator and denominator by the greatest common factor.** In the fraction at the right, the greatest common factor is 7 (the largest number that goes into both 14 and 63).

Step 2 **Check to make sure the fraction has been reduced completely!** As stated above, in a reduced fraction, only 1 can evenly divide both the numerator and denominator.

In the example above, the only number that can evenly divide both 2 and 9 is 1.

PRACTICE

Reduce the following fractions.

1) $\frac{6}{72} =$ 2) $\frac{10}{100} =$ 3) $\frac{3}{25} =$ 4) $\frac{6}{42} =$

ANSWERS

1) $\frac{1}{12}$ 2) $\frac{1}{10}$ 3) $\frac{3}{25}$ fraction cannot be further reduced 4) $\frac{1}{7}$

TYPES OF FRACTIONS

A. PROPER FRACTIONS

A fraction is proper if the numerator is smaller than the denominator.

PROPER FRACTIONS

$\frac{5}{25}$ $\frac{6}{13}$ $\frac{1}{5}$

B. IMPROPER FRACTIONS

A fraction is improper if the numerator is larger than (or equal to) the denominator.

IMPROPER FRACTIONS

$\frac{75}{25}$ $\frac{37}{13}$ $\frac{5}{5}$

C. MIXED NUMBERS (also called mixed fractions)

A mixed number contains a whole number and a fraction.

MIXED NUMBERS

$2\frac{4}{27}$ $3\frac{5}{9}$

CONVERTING FRACTIONS

To convert an improper fraction to a mixed number:

EXAMPLE

$\frac{37}{13} \Rightarrow 13\overline{)37}$ quotient 2, remainder 11

Step 1 **Divide the numerator by the denominator.**

Step 2 **Write the answer as a mixed number: In the example, the quotient 2 becomes the whole number and the remainder 11 becomes the numerator in the fraction. The denominator stays the same.**

$2\frac{11}{13}$

Step 3 **If necessary, reduce the fraction to lowest terms.**

To convert a mixed number to an improper fraction:

EXAMPLE

$4\frac{5}{6}$

Step 1 **Multiply the denominator by the whole number and then add the product to the numerator**.

$4 \times 6 = 24 \Rightarrow 24 + 5 = 29$

Step 2 **Place the sum over the denominator.**

$\frac{29}{6}$

PRACTICE

Convert the following improper fractions to mixed numbers.

1) $\frac{13}{8} =$ 2) $\frac{25}{7} =$ 3) $\frac{54}{7} =$ 4) $\frac{16}{5} =$

Convert the following mixed numbers to improper fractions.

5) $2\frac{5}{9} =$ 6) $13\frac{2}{3} =$ 7) $9\frac{1}{5} =$ 8) $11\frac{6}{7} =$

ANSWERS

1) $1\frac{5}{8}$ 2) $3\frac{4}{7}$ 3) $7\frac{5}{7}$ 4) $3\frac{1}{5}$ 5) $\frac{23}{9}$ 6) $\frac{41}{3}$ 7) $\frac{46}{5}$ 8) $\frac{83}{7}$

ADDING and SUBTRACTING FRACTIONS

When adding or subtracting fractions:

Step 1 **Convert mixed numbers to improper fractions.**

EXAMPLES

	ADDITION	SUBTRACTION
Step 2 **Find the lowest common denominator for the fractions. (A common denominator is a number that can be evenly divided by both denominators.)**	$\frac{1}{2} + \frac{3}{8} = ?$ 8 is the lowest common denominator	$\frac{2}{3} - \frac{3}{7} = ?$ 21 is the lowest common denominator
Step 3 **Convert each of the original fractions to equivalent fractions using the lowest common denominator.**	$\frac{4}{8} + \frac{3}{8} = ?$	$\frac{14}{21} - \frac{9}{21} = ?$
Step 4 **Add (or subtract) the numerators. Do not add or subtract the denominators.**	$\frac{4}{8} + \frac{3}{8} = \frac{7}{8}$	$\frac{14}{21} - \frac{9}{21} = \frac{5}{21}$

(Please note that the method described above is just one of several methods used in the addition and subtraction of fractions.)

PRACTICE

1) $\frac{3}{7} - \frac{2}{7} =$

2) $2\frac{9}{16} - 1\frac{5}{8} =$

3) $\frac{3}{7} + \frac{1}{2} + 2\frac{3}{14} =$

4) $\frac{7}{5} + \frac{8}{9} =$

5) $\frac{5}{6} + 2\frac{1}{3} =$

ANSWERS

1) $\frac{1}{7}$

2) $\frac{15}{16}$

3) $3\frac{1}{7}$

4) $\frac{103}{45} = 2\frac{13}{45}$

5) $\frac{19}{6} = 3\frac{1}{6}$

MULTIPLYING FRACTIONS

When multiplying fractions:

EXAMPLE

$1\frac{4}{9} \times \frac{2}{3} \times 1\frac{1}{7} \times 5$

Rewrite whole numbers as fractions: $5 = \frac{5}{1}$

Step 1 **Convert mixed numbers to improper fractions.**

Step 2 **Cancel (if possible).**

$\frac{13}{9} \times \frac{2}{3} \times \frac{8}{7} \times \frac{5}{1}$

↓

Step 3 **Multiply straight across. In other words, multiply all numerators, then all denominators.**

$\frac{13 \times 2 \times 8 \times 5}{9 \times 3 \times 7 \times 1} = \frac{1040}{189}$

↓

Step 4 **Simplify your answer. If you did not cancel before multiplying, you will need to reduce the answer.** Improper fractions are generally converted to mixed numbers for the final answer.

$\frac{1040}{189} = 5\frac{95}{189}$

PRACTICE

1) $\frac{3}{8} \times \frac{5}{6} =$

2) $\frac{2}{7} \times \frac{5}{4} =$

3) $\frac{52}{7} \times \frac{5}{14} =$

4) $3\frac{3}{7} \times \frac{13}{5}$

5) $\frac{8}{9} \times \frac{18}{24}$

6) $\frac{4}{8} \times \frac{5}{6} =$

7) $\frac{4}{3} \times 1\frac{5}{6} =$

8) $\frac{12}{2} \times 6 =$

9) $1\frac{4}{5} \times \frac{7}{5}$

10) $\frac{21}{9} \times \frac{9}{7}$

ANSWERS

1) $\frac{5}{16}$ 2) $\frac{5}{14}$ 3) $2\frac{32}{49}$ 4) $8\frac{32}{35}$ 5) $\frac{2}{3}$ 6) $\frac{5}{12}$ 7) $2\frac{4}{9}$ 8) 36 9) $2\frac{13}{25}$ 10) 3

DIVIDING FRACTIONS

When dividing fractions:

Step 1 **Convert all mixed fractions to improper fractions. Convert whole numbers to fractions.**

$1\frac{6}{7} \div \frac{5}{9} \rightarrow \frac{13}{7} \div \frac{5}{9}$

divisor

Step 2 **Invert the divisor**.

$\frac{5}{9}$ becomes $\frac{9}{5}$

Step 3 **Change the division sign to multiplication.**

$\frac{13}{7} \times \frac{9}{5}$

Step 4 **Cancel (if possible).**

Step 5 **Multiply straight across**:

$\frac{13 \times 9}{7 \times 5} = \frac{117}{35} = 3\frac{12}{35}$

Step 6 **Reduce if needed.**

PRACTICE

1) $\frac{8}{9} \div \frac{3}{4}$

2) $\frac{6}{3} \div \frac{3}{6}$

3) $\frac{5}{7} \div \frac{4}{9}$

4) $\frac{9}{15} \div 1\frac{4}{5}$

5) $3\frac{5}{6} \div 6$

6) $2\frac{3}{5} \div \frac{7}{5}$

7) $\frac{15}{16} \div \frac{4}{4}$

8) $9\frac{3}{5} \div \frac{6}{25}$

ANSWERS

1) $1\frac{5}{27}$ 2) 4 3) $1\frac{17}{28}$ 4) $\frac{1}{3}$ 5) $\frac{23}{36}$ 6) $1\frac{6}{7}$ 7) $\frac{15}{16}$ 8) 40

PERCENTS

Percent means "per one hundred". % is the symbol used to indicate percent.

The meaning of 15% is 15 parts in 100 parts.

Percents can be written as fractions: $15\% = \frac{15}{100}$

There are several methods for solving percent problems. In this review, the proportion method is used. With the proportion method, percent problems are solved using equivalent fractions. (Review the section above on equivalent fractions.)

Percent problems are commonly written as word problems. The biggest challenge in solving percent problems is understanding the wording so that the equivalent fractions can be set up properly.

An example of a percent problem is: "What is X% of Y?" Another way of stating this problem is: How many parts out of Y are equal to X parts out of 100?

To set up equivalent fractions, follow these guidelines:

- **i) Numbers after the word "of" are <u>always</u> in the denominators**.
- **ii) One of the denominators will <u>always</u> be 100.**
- **iii) The number in front of the percent sign (%) always has a denominator of 100.**

So for the problem "What is X% of Y?", the equivalent fractions are set up as shown below:

Fraction 1		Fraction 2
$\frac{X}{100}$	=	$\frac{\text{parts}}{Y}$

<u>EXAMPLES</u>

A) What is 74% of 50?

Another way of stating this problem is: How many parts out of 50 are equal to 74 parts out of 100?

Step 1 Write the equivalent fractions. $\frac{74}{100} = \frac{\text{how many parts?}}{50}$

Step 2 Follow the rules for equivalent fractions:

a) Look at the two denominators and divide the larger by the smaller: $100 \div 50 = \mathbf{2}$

b) Divide the numerator by the same number: $\frac{74\ (\div 2)}{100\ (\div 2)} = \frac{\text{how many parts?}}{50} = \frac{37}{50}$

c) Write the equivalent fractions: $\frac{74}{100} = \frac{37}{50}$

Step 3 The problem is solved: 74% of 50 is 37.

B) 42 is what percent of 600?

Another way of stating this problem is: 42 parts out of 600 is equivalent to how many parts out of 100?

Step 1 Write the equivalent fractions: $\frac{42}{600} = \frac{\text{how many parts?}}{100}$

Step 2 Follow the rules for equivalent fractions:

a) Look at the two denominators; divide the larger by the smaller: $600 \div 100 = 6$

b) Divide the numerator by the same number: $\frac{42\ (\div 6)}{600\ (\div 6)} = \frac{\text{how many parts?}}{100} = \frac{7}{100}$

c) Write the equivalent fractions: $\frac{42}{600} = \frac{7}{100}$

Step 3 The problem is solved: 42% of 600 is 7% (remember that $\frac{7}{100} = 7\%$).

PRACTICE

1) Find 40% of 50

2) 20 is what % of 250?

3) What percent of 80 is 60?

4) 40 is 80% of what number?

5) 32% of _____ is 64

6) 35 is what % of 700

ANSWERS

1) 20 2) 8 3) 75 4) 50 5) 200 6) 5

SIGNED NUMBERS

When adding or subtracting two numbers with different signs:

Step 1 Subtract the smaller number from the larger number.

Step 2 Give the answer the sign of the larger number.

Please note: A signed number problem can be written in several different ways:

$9 - 2$
$-2 + 9$
$9 + (-2)$

} The answer is 7 for all three problems!

EXAMPLE

$4 + (-10) = ?$

10 minus 4 equals 6

The larger number in the problem is 10 and it has a negative sign. Therefore the answer will also have a negative sign.

$4 + (-10) = -6$

When multiplying or dividing two signed numbers:

a. the answer is positive if both numbers have the same sign.

b. the answer is negative if the numbers have different signs.

EXAMPLES

$(-9) \times (-6) = 54$
$(-10) \div (-2) = 5$

$(-5) \times (4) = -20$

$\frac{3}{5} \times \left(-\frac{4}{7}\right) = -\frac{12}{35}$

PRACTICE

1) $(-8) + 4 =$

2) $(-6) + (-3) =$

3) $14 + (-2) =$

4) $5 - 13 =$

5) $\frac{3}{4} \times (-7) =$

62) $15 \div (-5) =$

7) $(-8) \times (-5) =$

8) $\frac{2}{3} \div \left(-\frac{6}{5}\right) =$

ANSWERS

1) -4 2) -9 3) 12 4) -8 5) $-\frac{21}{4} = -5\frac{1}{4}$ 6) -3 7) 40 8) $-\frac{5}{9}$

EXPONENTS

Exponents show how many times a number (called the base) is multiplied times itself. A number with an exponent is said to be "raised to the power" of that exponent.

Example: Three raised to the power of two is written: 3^2

3 is the base and 2 is the exponent (or power)

$3^2 = 3 \times 3 = 9$

I. To add or subtract numbers with exponents:

Step 1 Find the value of each number with an exponent

Step 2 Add the values together:

EXAMPLE

$3^3 + 2^3 = ?$

$3^3 = 3 \times 3 \times 3 =$ **27** and $2^3 = 2 \times 2 \times 2 =$ **8**

$27 + 8 = 35$ Therefore: $3^3 + 2^3 = 35$

II. To multiply numbers with exponents:

i) If the bases are the same, add the exponents and then perform the operation.

EXAMPLE

$2^3 \times 2^2 = 2^5 = 32$

because

$(2 \times 2 \times 2) = 8$ and $(2 \times 2) = 4$

and

$8 \times 4 = 32$

ii) If the bases <u>are not</u> the same:

Step 1 Find the value of each number with an exponent.

Step 2 Multiply the values together.

EXAMPLE

$5^2 \times 3^3 = 675$

because

$(5 \times 5) = 25$ and $(3 \times 3 \times 3) = 27$

and

$25 \times 27 = 675$

<u>PRACTICE</u>

1) $3^3 \times 6^2 =$

2) $5^2 + 3 =$

3) $9 \div 3^2 =$

4) $5^3 \times 5^2 =$

5) $7^2 - 5^2$

6) $50 \div 5^2 =$

7) $9^2 + 4^2 + 3 =$

8) $3^3 \times 2^4 =$

<u>ANSWERS</u>

1) 972	2) 28	3) 1	4) 3125
5) 24	6) 2	7) 100	8) 432

PRE-TEST

If you can accurately solve the following 24 problems in 30 minutes without a calculator, you are ready for the entrance test. (On the actual test, you will have 1 hour to complete 48 questions.)

1) $2.627 - 0.2 =$

2) $58.25 + 19.112 =$

3) $180 \div 15 =$

4) $\frac{7}{9} \times 6 =$

5) $5\frac{2}{5} - 2\frac{3}{15}$

6) $13 \overline{)266.5}$

7) $\frac{4}{5} \div 3\frac{2}{7} =$

8) $195.18 \times 3 =$

9) $5.8 \overline{)73.08}$

10) $29.5 \div 100 =$

11) $3\frac{2}{5} \div 2\frac{2}{5} =$

12) $\frac{5}{8} - \frac{5}{24}$

13) $4\frac{1}{5} \times \frac{1}{3} =$

14) $2028 \div 39 =$

15) $2.248 \times 8 =$

16) $\frac{3}{10} + \frac{1}{4} + \frac{3}{5} =$

17) $8\frac{2}{5} + 1\frac{3}{7} =$

18) $29 - 3.7 =$

19) 25% of $\boxed{?} = 175$

20) $4^2 \times 4^4 =$

21) $-\frac{4}{5} - \frac{3}{5} =$

22) 0.53×0.42

23) $-18 \div \frac{3}{5} =$

24) $7 \overline{)36,425}$

Answers

1) 2.427	11) 1 5/12	21) –1 2/5
2) 77.362	12) 5/12	22) 0.2226
3) 12	13) 1 2/5	23) – 30
4) 4 2/3	14) 52	24) 5203 remainder 4
5) 3 1/5	15) 17.984	
6) 20.5	16) 1 3/20	
7) 28/115	17) 9 29/35	
8) 585.54	18) 25.3	
9) 12.6	19) 700	
10) 0.295	20) 4096	